

## EVOLUTION OF THE EXPANDING UNIVERSE

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The problem of the universe is essentially an application of the law of gravitation to a region of extremely low density. The mean density of matter up to a distance of some ten millions of light years from us is of the order of  $10^{-30}$  gr./cm.<sup>3</sup>; if all the atoms of the stars were equally distributed through space there would be about one atom per cubic yard, or the total energy would be that of an equilibrium radiation at the temperature of liquid hydrogen. The theory of relativity points out the possibility of a modification of the law of gravitation under such extreme conditions. It suggests that, when we identify gravitational mass and energy, we have to introduce a constant. Everything happens as though the energy *in vacuo* would be different from zero. In order that absolute motion, i.e., motion relative to vacuum, may not be detected, we must associate a pressure  $p = -\rho c^2$  to the density of energy  $\rho c^2$  of vacuum. This is essentially the meaning of the cosmical constant  $\lambda$  which corresponds to a negative density of vacuum  $\rho_0$  according to

$$\rho_0 = \frac{\lambda c^2}{4\pi G} \cong 10^{-27} \text{ gr./cm.}^3 \quad (1)$$

Let us consider the motion of matter symmetrically distributed round some fixed point 0. The classical equation of motion under the action of the modified gravitational field is

$$\left(\frac{dr}{dt}\right)^2 = -h + \frac{2Gm}{r} + \frac{\lambda c^2}{3} r^2 \quad (2)$$

where  $m$  is the mass inside the sphere of radius  $r$  and center 0. The condition that the system expands, remaining similar to itself, is that  $h$  and  $m$  have to be proportional, respectively, to  $r^2$  and  $r^3$ . This classical motion is a good approximation of the relativistic equations when  $r$  is small enough. When  $r$  is great, some geometrical modifications become important and the classical model must be interpreted as a map in euclidean space. This map is like an orthogonal projection: lengths perpendicular to the radius vector are not altered, but along the radius vector they are represented at a scale

$$\sqrt{1 - h/c^2} \quad (3)$$

and the scale vanishes at the boundary of the map where  $h = c^2$ . If we write

$$h = c^2 \sin^2 \chi, \quad r = R(t) \sin \chi, \quad (4)$$

we obtain Friedmann's equation, the variable radius  $R(t)$  of the map being the so-called radius of space. Antipodal points at the boundary of the map are supposed to represent the same real points. Then it can be proved that if we change the center 0 of the representation the representation remains exactly the same.

The model we have described, we called the idealized model; it is perfectly homogeneous. If there are some fluctuations of density and velocity round the mean value, we can continue to apply classical mechanics (with the modified gravitational law) when we restrict ourselves to a domain not too large in respect to the whole volume of space. Let us suppose that the motion in the idealized model is of the ever-expanding type: a retarded expansion passing through a minimum velocity at some time  $t_c$  when the total gravitation force vanishes, and expanding again under the predominant effect of the cosmical repulsion, the velocity tending finally to infinity according to the law

$$\frac{dr}{dt} = r \sqrt{\frac{\lambda c^2}{3}} = r \sqrt{\frac{4\pi}{3} G \rho_0}. \quad (5)$$

For the perturbed motion, i.e., for a distribution of mass and initial velocities somewhat different from the idealized model, the motion at some places may be of a completely different type from the motion of the idealized model. The relation between the energy-constant  $h$  and the mass  $m$  may be such that the motion is of the collapsing type: the expansion velocity vanishes when the gravitation is not yet completely balanced by the cosmical repulsion and the expansion is followed by a contraction. The result of the perturbations is that, after the time  $t_c$ , the system includes collapsing regions, distributed in the generally expanding space. That means that we obtain collapsing regions flying away one from another with velocities roughly proportional to the distance.

Occasionally, we may also have equilibrium-regions. The fact that such an equilibrium is unstable means only that it will occur relatively rarely and that collapsing regions will be decidedly more frequent than equilibrium-regions. Furthermore the equilibrium cannot be a detailed equilibrium, so that an equilibrium-region must divide itself into collapsing regions, and these collapsing regions will remain approximately at the same distance one from another.

The hypothesis we wish to discuss is that collapsing regions must be identified with the extra-galactic nebulae and the equilibrium-regions with the clusters of nebulae.

This hypothesis implies that the mean density in the clusters of nebulae must be the same for all, and furthermore must be connected with Hubble's

ratio of distance to spectroscopic velocities by the approximate relation (5). The epulilibrium-regions must have quite irregular forms, just as the clusters of nebulae have. For spherical form, we must have the relation

$$Nm = C D^3 d^3 \quad (6)$$

where  $N$  is the total number of nebulae in the cluster,  $m$  the mean mass of a nebula for which we choose as a unit  $10^9$  times the mass of the sun,  $D$  the distance of the cluster in mega-parsecs,  $d$  the angular diameter in degrees. The constant  $C$  depends on the velocity  $V$  in thousand km./sec. of the nebulae at a distance of a mega-parsec by the relation

$$C = \frac{3.084}{8 \times 6.664 \times 1.983 \times (0.573)^3} V^2 = 0.155 V^2. \quad (7)$$

In a previous paper,<sup>1</sup> I have compared the hypothesis of equilibrium with Hubble and Humason's data on eight clusters of nebulae.<sup>2</sup> In Hubble's work, the frequency distribution is determined and the distance deduced from the most frequent magnitude and checked by velocity determinations. This comparison is reproduced with some changes, explained later on, in table 1. Simultaneously with the publication of

TABLE 1

CLUSTER	N	D	d	m
Coma	800	19	1.7	1.1
Perseus	500	15	2.0	1.4
Leo	400	46	0.6	1.3
Urs. Maj.	300	30	0.7	0.8
Cancer	150	13	1.5	1.1
Pegasus	100	10	1	0.3

my paper quoted above, new data were published by Shapley<sup>3</sup> concerning 25 other groups of galaxies. The luminosity curve includes only the brightest members of the cluster and we can get an estimate of the total population and mean magnitude by identifying Shapley luminosity curves with the brightest part of the mean luminosity curve determined by Hubble. This mean luminosity curve is essentially an equilateral triangle of basis five magnitudes. We therefore try to represent Shapley luminosity curves by a straight line  $a(m - b)$  and take as estimated population  $169 a/5$  and mean magnitude  $b + 2.5$ . This process gives very definite results for nine of the Shapley groups. They are tabulated in table 2. For nine other groups the process can be applied but with serious uncertainty as the fluctuations round the straight line are fairly large (table 3). For seven of the poorest groups the distribution has no appearance of similitude with Hubble's frequency curve and we have been obliged to neglect them. We have included under  $n$  the observed number of nebulae

in Shapley's restricted area and taken for  $d$  the angular diameter corresponding to his  $\sin \theta$ . In the computation of both Hubble's and Shapley's data, we have used, following Shapley and Knox Shaw,<sup>3</sup>  $-14.5$  as mean photographic magnitude of a nebula and  $405 \text{ km./sec.}$  for the spectroscopic velocity at a megaparsec.

Except for a systematic difference between Hubble's and Shapley's data, the constancy of the computed  $m$  is very remarkable. This systematic difference corresponds to a change of  $0.8$  in the system of magnitude. This is what we may expect as Hubble and Shapley find, respectively,  $13$  and  $14$  as the most frequent magnitude of the Virgo cluster.

TABLE 2

GROUP	n	N	D	d	m
14	370	3800	46	1.57	2.0
20	341	1500	46	1.50	5.6
8	317	2700	44	1.67	3.7
15	256	3200	50	1.23	1.8
17	157	940	55	1.00	4.4
13	150	1300	46	1.17	2.9
3	77	270	44	0.67	2.4
25	56	340	63	0.53	2.8
4	46	340	50	0.67	2.7

TABLE 3

GROUP	n	N	D	d	m
16	90	340	44	0.87	4.1
2	88	170	30	1.00	4.0
23	82	300	44	1.00	7.1
21	66	240	44	0.60	1.9
10	58	170	36	0.83	4.0
22	58	300	44	0.50	8.8
5	56	90	27	0.50	0.7
7	42	150	33	0.50	7.7
11	45	150	45	0.60	3.3

We therefore find from the value of the red-shift and data on the clusters a mean mass of a nebula of one or three  $10^9$  suns. This is the order of magnitude that was deduced by Hubble from the rotation and absolute magnitude of some bright nebulae.<sup>4</sup>

It might be noticed that our determination includes obscure matter. We can therefore conclude that, if our hypothesis can be accepted, the uncertainty factor of  $100$  or  $1000$  which is generally ascribed to Hubble's determination of the mean mass of a nebula must be considered as greatly exaggerated.

We have seen that our identification of the clusters of nebulae with equilibrium-regions is substantiated by observation. We must now inquire under what conditions we can identify the nebulae themselves

with the collapsing regions. A nebula of  $10^9$  suns comes out of a collapsing region of initial volume corresponding to a radius  $r_c$  given by

$$\frac{4\pi}{3} \rho_0 r_c^3 = m \quad (8)$$

that is, about 100,000 light years. For the radius of a normal nebula, we can take 1000 light years. This difference accounts for the degree of concentration observed in the nebulae. But what happens to the gravitational energy due to the contraction?

If, for definiteness, we consider the distribution of matter in a nebula as a polytrop of index 3, this energy is

$$\frac{3}{2} G \frac{N^2 m^2}{R} \quad (9)$$

where  $R$  is the radius of the nebula and  $N$  the number of stars of mass  $m$ . For comparison the gravitational energy of one star is

$$\frac{3}{2} G \frac{m^2}{r} \quad (10)$$

where  $r$  is the radius of the star. Therefore the loss of energy is  $Nr/R$  times the gravitational energy of the stars. For  $N = 10^9$ ,  $r = 6 \times 10^{10}$  cm. (radius of the sun) and  $R = 1000$  light years  $= 10^{21}$  cm. we find that we have to account for a loss of energy of the order of 6 per cent of the gravitational energy of the stars in the nebula. If the stars existed as stars before the critical instant when the collapse began, there would be no way to account for such a loss of energy. But, if before the critical instant matter is formed of gas, dust or meteorites of comparatively small free path, the collapse will produce a number of non-elastic collisions, turning out gravitational energy into heat, and agglomerating the diffuse matter into large hot masses, i.e., into stars.

We are therefore led to the conclusion that, in the frame of our hypothesis, the same mechanism which provides the formation of the nebulae, provides also the formation of the stars. Stars and nebulae are formed by the same process, and there is no star which is not associated with a nebula.

Another consequence is that obviously the total volume of the expanding and of the collapsing or equilibrium regions at the critical instant cannot be of different order of magnitude. It follows that the total mass of obscure matter in inter-nebular space is of the same order of magnitude as the total mass agglomerated into nebulae, i.e.,  $10^{-30}$  gr./cm.<sup>3</sup>

The difference of types of the nebulae may be accounted for as a difference of the total angular momentum of the corresponding collapsing

regions. Finally, we must expect the occurrence of an intermediary type between the collapsing regions and the equilibrium regions. There must exist slowly collapsing regions containing a number of rapidly collapsing regions, and this could be identified with our galaxy.

We may expect to get a complete theory of all the problems connected with extra-galactic nebulae by applying statistical mechanics to small inhomogeneity in our idealized model. Such an investigation would probably involve only two parameters; one to fix the mean velocity of expansion at the instant of equilibrium, a second one to define the dispersion of the distribution of matter from the idealized model.

<sup>1</sup> *Ann. Société Scientifique Bruxelles*, Série A 53, p. 51-85 (1933). C. R. Paris, March 24 and April 10 (1933).

<sup>2</sup> Mt. Wilson Contribution No. 427.

<sup>3</sup> *Proc. Nat. Acad. Sci.*, 19, 591-596 (1933).

<sup>4</sup> Mt. Wilson Contribution No. 324.

## AN INVESTIGATION OF THE STRESSES IN LONGITUDINAL WELDS<sup>1</sup>

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1. During the last four years a theoretical and experimental study of the stresses in longitudinal welds has been carried out at the Massachusetts Institute of Technology. The early part of this work has already been reported in these PROCEEDINGS, 16, pp. 667-678 (1930) and 17, pp. 351-359 (1931); the later part consists of three theses and a great amount of further study. The research had for its main object the determination of the stresses in a longitudinal weld as well as in the adjoining structural members, and centered in the simple and fundamental case where a long rectangular plate, subject to a lengthwise pull, is reinforced by a double flat bar or web connected to it by four fillet welds.

2. The problem was first attacked by the author, as described in the papers referred to above, by assuming that the shearing stress at any point in the weld is proportional to the average displacement of the bar relative to the plate across a transverse section through that point. Expressed in symbols:

$$q_x = U_x/\mu \quad (1)$$

where  $q_x$  is the shearing stress on the throat area of the weld.  $U_x$  is the relative displacement and  $\mu$  is a coefficient, hereafter referred to as the